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Complete photonic bandgaps for all polarizations in one-dimensional photonic crystals

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Abstract
The photonic band structures of one-dimensional photonic crystals (1D PCs) with three layers in a period are studied theoretically. Our results show that complete photonic bandgaps (PBGs) for all polarizations can be obtained in these 1D PCs made of a conventional dielectric material, a negative-permittivity and a negative-permeability metamaterial. The origin of the complete PBGs lies in the existence of surface waves for all polarizations.

1. Introduction

Photonic crystals (PCs) are composite materials with a spatially periodic variation of permittivity and/or permeability [1–3]. As a result of multiple Bragg scatterings, PCs are characterized by complicated photonic band structures. Between photonic bands there may exist a photonic bandgap (PBG) for frequency within which the propagation of electromagnetic waves is absolutely forbidden. The existence of complicated photonic band structures and PBGs in PCs offers excellent approaches in the control of the dispersion and propagation of electromagnetic waves, potentially leading to many applications in photonic devices.

A PBG for all propagation directions (termed a complete PBG) and polarizations is highly desired for many applications. For dielectric constituents, one-dimensional (1D) and 2D PCs do not possess such complete PBGs. This is due to the fact that in dielectric PCs the formation of PBGs is determined by the Bragg condition simultaneously in all propagation directions. Complete PBGs can be obtained only if waves reflected at different interfaces interfere constructively for all propagation directions. As a result, only 3D dielectric PCs may possess complete PBGs for all polarizations. Compared with 2D and 3D PCs, the design and fabrication of 1D PCs are much more amenable. Thus, the searching for 1D PCs with complete PBGs for all polarizations is of great significance.

In dielectric 1D PCs, only partial PBGs exist for certain propagation directions. In a recent paper [4], Shadrivov *et al* studied 1D PCs consisting of left-handed metamaterials [5–7]. They showed theoretically that complete PBGs for one polarization can exist in 1D PCs consisting of two layers in a period made of a conventional dielectric material and a left-handed metamaterial. They further suggested that a 1D PC consisting of three layers in a period made of a conventional dielectric material and two left-handed metamaterials could trap light in three dimensions due to the existence of complete PBGs for all polarizations. This interesting finding could render possible novel applications of metamaterials in a wide wavelength range.

In this paper, we show that 1D PCs consisting of three layers in a period made of a conventional dielectric material, a negative-permittivity and a negative-permeability metamaterial [8–10] can possess complete PBGs for all polarizations. The origin of these PBGs relies on the existence of surface waves for all polarizations.

2. Formalism for photonic band structures

We consider 1D PCs consisting of three layers in a period. The permittivity and permeability of the constituents are denoted respectively by ε_i and μ_i, where the subscript $i = 1, 2, 3$ stands for the layer index. Layer thickness is denoted by d_i.

Thus, the period of 1D PCs is given by $a = d_1 + d_2 + d_3$. Using a transfer matrix method and imposing the Bloch theorem [11, 12], it can be shown the photonic band structures of 1D PCs with three layers in a period can be obtained from the following dispersion relation:

$$\cos(qa) = c_1c_2c_3 - \frac{1}{2}\left(\frac{\eta_1}{\eta_2}+\frac{\eta_2}{\eta_1}\right)s_1s_2c_3 - \frac{1}{2}\left(\frac{\eta_2}{\eta_3}+\frac{\eta_3}{\eta_2}\right)c_1s_2s_3 - \frac{1}{2}\left(\frac{\eta_1}{\eta_3}+\frac{\eta_3}{\eta_1}\right)s_1c_2s_3, \quad (1)$$

where $c_i = \cos(k_{iz}d_i)$ and $s_i = \sin(k_{iz}d_i)$. Here, the periodic direction is assumed to be along the z direction, the local wavevector of the ith layer $k_i = \sqrt{\varepsilon_i\mu_i}\omega/c$ is assumed to lie in the xz plane, namely, $\mathbf{k}_i = (k_{ix}, 0, k_{iz})$ and q is the Bloch wavevector of 1D PCs in the first Brillouin zone. Equation (1) is hold for both s (with the electric field perpendicular to the periodic direction) and p (with the magnetic field perpendicular to the periodic direction) polarizations. The only difference is the parameters η_i: $\eta_i = k_{iz}/\varepsilon_i$ for p polarization and $\eta_i = k_{iz}/\mu_i$ for s polarization. It can be easily seen that, if $d_3 = 0$, equation (1) is reduced to

$$\cos(qa) = c_1c_2 - \frac{1}{2}\left(\frac{\eta_1}{\eta_2}+\frac{\eta_2}{\eta_1}\right)s_1s_2, \quad (2)$$

which is known to be the dispersion relation for 1D PCs with two layers in a period [11].

3. Results and discussions

1D PCs under study consist of three layers in a period made of a conventional dielectric material, a negative-permittivity metamaterial and a negative-permeability metamaterial [8–10]. Without loss of generality, the first layer is assumed to be a dielectric material with $\varepsilon_1 = 2.25$ and $\mu_1 = 1$. The second layer is a negative-permittivity metamaterial with

$$\varepsilon_2 = 1 - \omega_p^2/\omega^2, \qquad \mu_2 = 1, \quad (3)$$

and the third layer is a negative-permeability metamaterial with

$$\mu_3 = 1 - \omega_c^2/\omega^2, \qquad \varepsilon_3 = 1, \quad (4)$$

where ω_p and ω_c are resonant frequencies of the metamaterials. It is found that complete PBGs for all polarizations are strongly dependent on the ratio ω_p/ω_c. When the difference between ω_p and ω_c is small, it is more amiable to open up large complete PBGs for all polarizations. Thus, in the following discussion we assume $\omega_p/\omega_c = 1$.

For an arbitrary propagating direction, a 1D PC is periodic in the z direction. But it is homogeneous in the xy plane. In a structure with an infinite number of layers, for both s and p polarizations, the fields must be Bloch wave solutions due to the translational symmetry along the in-plane, namely

$$E_q(x, z) = E_q(z)e^{iqz}\, e^{ik_x x}, \quad (5)$$

where $E_q(z)$ is a periodic function of the lattice constant a. The Bloch wavevector q is the solution of equation (1) and can be real or imaginary, corresponding to propagating or evanescent waves.

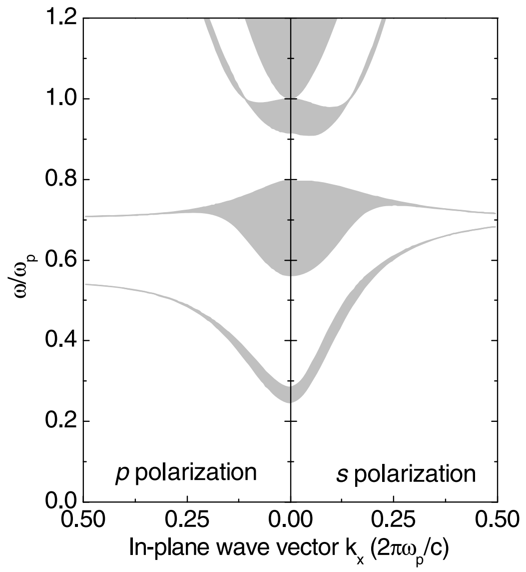

Figure 1. Projected photonic band structures for a 1D PC consisting of a dielectric layer, a metamaterial layer with single negative-permittivity and a metamaterial layer with single negative-permeability. The thicknesses of the three layers are $d_1 = 2c/\omega_p$ and $d_2 = d_3 = c/\omega_p$. Gray areas denote photonic bands.

For an arbitrary direction of propagation, it is better to examine the projected photonic band structures over the in-plane wavevector k_x (k_x should be identical at each layer due to the constraint of the boundary conditions). Since along the z direction 1D PCs are periodic, q should be restricted to the range $0 \leqslant q \leqslant \pi/a$. The allowed mode frequencies $\omega_n(q, k_x)$ for each choice of q constitute the projected photonic band structures, where n is the band index. From the projected photonic band structures we can easily determine whether a 1D PC possesses complete PBGs or not.

Figure 1 shows the projected photonic band diagram of a 1D PC with three layers in a period over the in-plane wavevector k_x. Obviously, there are two complete PBGs for either s or p polarization: one between the upper edge of the second photonic band and the lower edge of the third photonic band and the other below the lower edge of the first photonic band. For frequency above ω_p, no complete PBGs for both s and p polarizations exist. For both s and p polarizations, the low frequency complete PBG is the same, below a cutoff frequency $0.247\omega_p$, similar to metals with a cutoff plasma frequency. For s polarization, the high frequency complete PBG ranges in frequency from $0.797\omega_p$ to $0.909\omega_p$. For p polarization, it is a bit different, spanning in frequency from $0.797\omega_p$ to $0.915\omega_p$. Thus, the high frequency complete PBG for all polarizations should be an overlapping range of the two, namely from $0.797\omega_p$ to $0.909\omega_p$. Within these two complete PBGs, the photonic density of states should be zero since there are no photonic bands. As a result, the propagation of electromagnetic waves with either s or p polarization is not allowed for frequencies within these complete PBGs.

It is interesting to note that for s polarization the first and second photonic bands approach asymptotically the same

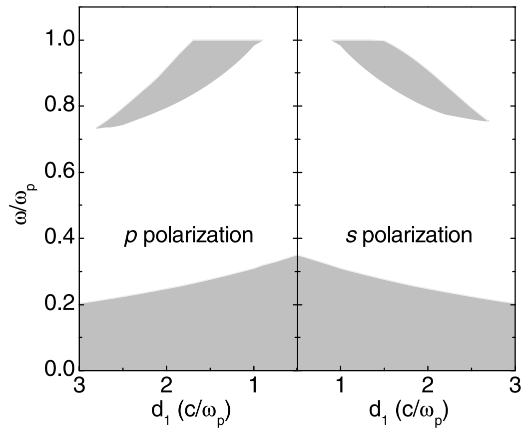

Figure 2. Gap map as a function of d_1 while $d_2 = d_3 = c/\omega_\mathrm{p}$ is kept fixed. The shaded regions denote complete PBGs.

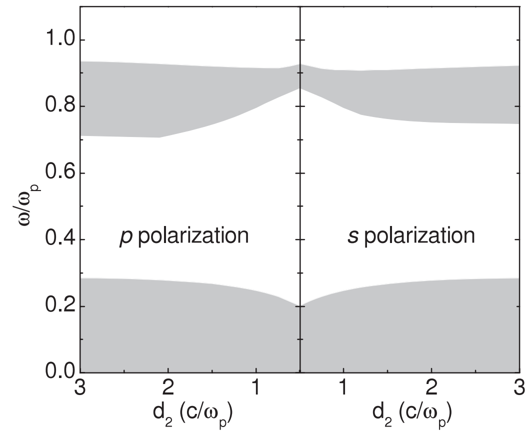

Figure 3. Gap map as a function of d_2 while $d_1 = 2c/\omega_\mathrm{p}$ and $d_3 = c/\omega_\mathrm{p}$ is kept fixed. The shaded regions denote complete PBGs.

frequency for large in-plane wavevectors. For p polarization, they are asymptotic to different frequencies. This is due to the existence of surface waves, similar to surface plasmon-polaritons at metal/dielectric interfaces [13]. For an interface formed by two materials, surface waves may exist for both s and p polarizations if one material is a metamaterial. This is different for metal/dielectric interfaces: only p polarization can sustain surface plasmon-polaritons. It can be shown that the general dispersion relation for surface waves at an interface can be obtained from the following relation:

$$\frac{k_{1z}}{\alpha_1} + \frac{k_{2z}}{\alpha_2} = 0, \tag{6}$$

where $\alpha = \varepsilon$ for p polarization and $\alpha = \mu$ for s polarization. If the above equation has solutions the interface can sustain surface waves. For large in-plane wavevectors (the nonretarded condition), surface waves approach asymptotically a resonant frequency, obtained from $\alpha_1 + \alpha_2 = 0$.

For s polarization, only the interface between the dielectric and negative-permeability metamaterial can sustain surface waves. The corresponding resonant frequency is $\omega_\mathrm{p}/\sqrt{\mu_1 + 1}$. Due to the s polarized surface waves, the first and second photonic bands approach asymptotically this resonant frequency. For p polarization, both the interface between the dielectric and negative-permittivity metamaterial and the interface between the negative-permittivity and negative-permeability metamaterials can sustain surface waves. For the interface between the dielectric and negative-permittivity metamaterial, the resonant frequency is $\omega_\mathrm{p}/\sqrt{\varepsilon_1 + 1}$. For the interface between the negative-permittivity and negative-permeability metamaterials, the resonant frequency is $\omega_\mathrm{p}/\sqrt{\varepsilon_3 + 1}$. As a result, the first and second photonic bands approach asymptotically these two resonant frequencies separately.

The position and width of PBGs can be tuned by changing the thickness of the constituent layers. Figure 2 shows the gap map as a function of d_1 while $d_2 = d_3 = c/\omega_\mathrm{p}$ is kept fixed. For both s and p polarizations, the first complete PBGs at low frequency overlap each other completely. The width of these PBGs decreases with increasing d_1. For p polarization, the second complete at high frequency can only exist for d_1 in the range between $0.95c/\omega_\mathrm{p}$ and $2.8c/\omega_\mathrm{p}$, while for s polarization it exists for d_1 in the range between $0.95c/\omega_\mathrm{p}$ and $2.7c/\omega_\mathrm{p}$. This complete PBG for all polarizations is determined by that of s polarization due to the fact that the complete PBG of s polarization is completely within that of p polarization.

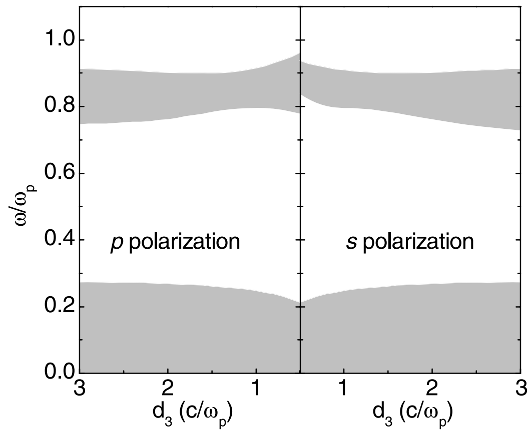

Figure 4. Gap map as a function of d_3 while $d_1 = 2c/\omega_\mathrm{p}$ and $d_2 = c/\omega_\mathrm{p}$ is kept fixed. The shaded regions denote complete PBGs.

In figure 3 we show the gap map as a function of d_2 while $d_1 = 2c/\omega_\mathrm{p}$ and $d_3 = c/\omega_\mathrm{p}$ are kept unchanged. The complete PBG at low frequency is the same for both s and p polarizations. Its width increases with increasing d_2 and tends to a constant for large values of d_2. For both s and p polarizations, the complete PBG at high frequency has a similar behavior. The complete PBG of s polarization is totally within that of p polarization. Thus, this complete PBG for all polarizations is determined by that of s polarization.

Figure 4 shows the gap map as a function of d_3 while $d_1 = 2c/\omega_\mathrm{p}$ and $d_2 = c/\omega_\mathrm{p}$ are kept unchanged. The first PBG at low frequency is identical for both s and p polarizations. Its width increases with increasing d_3 and tends to a constant for large values of d_3. For small d_3, the complete PBG for all polarization at high frequency is determined by that of s polarization. However, for larger d_3 it is determined by the complete PBG of p polarization. The width of this PBG for all polarization tends to a constant for large d_3.

From the above discussions, it can be found that the origin of complete PBGs in 1D PCs with three layers resides in the existence of surface waves. The photonic bands related to surface waves should be asymptotic to the surface wave resonant frequencies at large in-plane wavevectors. Consequently, complete PBGs may open up. Complete PBGs for all polarizations may exist if the interfaces can sustain surface waves for both s and p polarizations. In contrast, if 1D PCs do not sustain any surface wave, photonic bands should increase their frequencies with increasing in-plane wavevector. As a result, no complete PBG is expected.

For practical considerations, the thickness of the metamaterials should be much larger than their characteristic dimensions such that they can be viewed as homogeneous effective media. From our results, it can be found that our three-layer PCs possess a promising feature: the width of both complete PBGs increases with increasing thickness of metamaterials. We can thus choose thicker metamaterials in order to satisfy the homogenization requirement.

4. Conclusion

The photonic band structures of 1D PCs consisting of three layers in a period made of a conventional dielectric material, a negative-permittivity and a negative-permeability metamaterial are studied theoretically. Our results show that two complete PBGs exist for all polarizations. The position and width of these complete PBGs can be tuned by changing the thickness of the constituent layers. The existence of complete PBGs for all polarizations implies that these 1D PCs can trap photons of all polarizations in three dimensions.

Acknowledgments

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